QUIZ 23 SOLUTIONS: LESSON 29
APRIL 5, 2019

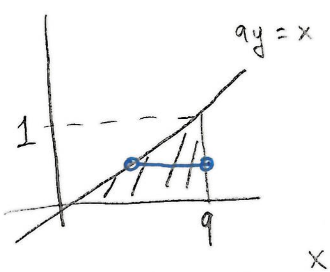
Write legibly, clearly indicate the question you are answering, and put a box or circle around your final answer. If you do not clearly indicate the question numbers, I will take off points. Write as much work as you need to demonstrate to me that you understand the concepts involved. If you have any questions, raise your hand and I will come over to you.

1. [5 pts] Compute

$$\int_0^1 \int_{9y}^9 e^{x^2}\, dx\, dy.$$

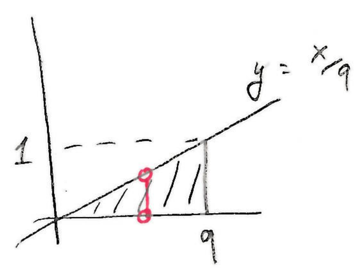
We cannot integrate $\int e^{x^2} dx$, so we need to swap the order of integration.

$$R = \begin{cases} 9y \le x \le 9 \\ 0 \le y \le 1 \end{cases} \longrightarrow R: \begin{cases} 0 \le y \le \frac{x}{9} \\ 0 \le x \le 9 \end{cases}$$



$9y = x \iff y = x/9$

$9y \le x \le 9$



$0 \le y \le x/9$

$$\int_0^1 \int_{9y}^9 e^{x^2} dx\, dy = \int_0^9 \int_0^{x/9} e^{x^2} dy\, dx$$

$$= \int_0^9 y e^{x^2} \Big|_{y=0}^{y=\frac{x}{9}} dx$$

$$= \int_0^9 \left(\frac{x}{9}\right) e^{x^2} dx$$

$u = x^2$

$du = 2x\,dx$

$\frac{du}{2x} = dx$

$$= \int_{u(0)}^{u(9)} \left(\frac{x}{9}\right) e^{u} \left(\frac{du}{2x}\right)$$

$$= \int_{u(0)}^{u(9)} \frac{1}{18} e^u du$$

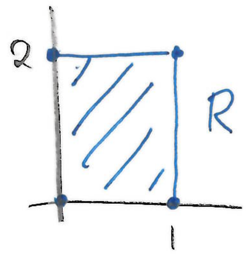
$$= \frac{1}{18} e^u \Big|_{u(0)}^{u(9)}$$

$$= \frac{1}{18} e^{x^2} \Big|_0^9$$

$e^0 = 1$

$$= \boxed{\frac{1}{18} e^{81} - \frac{1}{18}}$$

2. [5 pts] Find the average value of $f(x, y) = 7x^2$ over the rectangle with vertices $(0, 0)$, $(1, 0)$, $(0, 2)$, $(1, 2)$.



Area $(R) = 2$

$$R = \begin{cases} 0 \le y \le 2 \\ 0 \le x \le 1 \end{cases}$$

For $f(x,y)$ a function, its average value over a region R is
$$\text{Ave}_f = \frac{1}{\text{Area}(R)} \iint_R f(x,y)\, dA$$

So,

$$\begin{aligned}
\text{Ave}_f &= \frac{1}{2} \int_0^1 \int_0^2 7x^2 \, dy\, dx \\
&= \frac{1}{2} \int_0^1 7x^2 y \Big|_{y=0}^{y=2} dx \\
&= \frac{1}{2} \int_0^1 14x^2 \, dx \\
&= 7 \int_0^1 x^2 \, dx \\
&= 7 \left(\frac{1}{3}\right) x^3 \Big|_{x=0}^{x=1} \\
&= \boxed{\frac{7}{3}}
\end{aligned}$$